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Lessons Learned From Detracked Mathematics Departments

Students' mastery of and achievement in high school mathematics is considered pivotal to their opportunities for and within postsecondary education. For this reason, many educators have attempted to implement equity-geared reforms, including detracking, that affect the organization and instruction of high school mathematics. This article describes how schools with successful detracked mathematics programs share 4 characteristics: (a) a view of subject that focuses on connections and meaning, (b) a curriculum focused on important mathematical ideas, (c) a balance of coordination and professional discretion for teaching decisions, and (d) clear distinctions between doing math *and* doing school *in the structures of the classroom and the evaluation of students' thinking. This analysis can support other schools' attempts to detrack mathematics.*

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MATHEMATICS IS AN ACADEMIC DOMAIN often perceived as beyond the reach of educational reforms. It is not uncommon for whole-school reforms, such as graduation portfolios or project-based learning, to take hold in every subject except mathematics (Horn, 2002; Little, 1999). This is due to the conventional wisdom that mathematics is unique among the disciplines in its lack of adaptability to more open-ended styles of teaching and learning. How can we teach mathematics for understanding, for example, if the subject is made up of discrete facts that need to be memorized? How can we support students' collaborative learning when successful learning is viewed as individuals mastering procedures?

Due to these widespread perceptions, even advocates of mixed-ability grouping and detracking often shy away from truly detracking high school mathematics departments—that is, reconstructing the curriculum so that students are grouped heterogeneously. Learning about what works is made more complicated by the multiple versions of detracking that exist (Rubin, 2003). Gutiérrez (1996) pointed out that detracking reforms are difficult to evaluate on a large scale due to their multiple local variations. In some cases, detracking may only refer to the elimination of ability grouping within a particular class. For example, instead of having college preparatory and regular sections of algebra, a school might detrack by just offering algebra. Elsewhere, it may refer to the elimination of noncollege-preparatory classes, such as general math, consumer math, or pre-algebra. Finally, detracking may refer to a more radical change, a curriculum structure in which all students enter the same college-preparatory math class in their first year of high school.

Due to this range in approaches to detracking, I focus this article on two well-documented cases of mathematics departments that detracked in the final mode. By focusing on two successful instances of full-fledged detracking, this article uncovers some of the organizational and conceptual factors that supported this success. This analysis may provide a provisionary guide for other attempts to detrack secondary mathematics.

Two Cases of Detracking

The first department, in an urban working-class British school called Phoenix Park[1] was extensively documented in a 3-year comparative study by Jo Boaler (2002a). Her book based on this study, *Experiencing School Mathematics,* serves as the primary source for this analysis of the teaching practices in that department. The second department was in a diverse urban U.S. school called East High School. I studied the East math teachers' work practices while teaching alongside them (Horn, 2002, 2004a, 2004b, 2005). Subsequent to my study of the teachers' work practices, Boaler conducted a 4-year longitudinal study of mathematics teaching and learning at the school, providing documentation of the positive student learning outcomes there (Boaler, this issue; Boaler, Brodie, & Shahan, 2004; Boaler & Staples, under review).[2] These studies serve as the primary sources for this article.

Both departments have been successful in decreasing the persistent inequities that have been found in mathematics classrooms (Oakes, 1990). That is, in these departments, students who belonged to groups traditionally underrepresented in higher levels of mathematics demonstrated higher achievement patterns and took advanced mathematics at greater rates than students in more conventional settings. In addition, students at both schools reported having more positive feelings about mathematics and of themselves as mathematics learners.[3]

In this article, I examine the specific practices and structures shared by both departments to better understand the factors that underlie their success. Within each of these commonalities, there were notable differences that might reveal the way that a reform like detracking can be adapted to particular contexts. In all, I found that the two departments shared four attributes: (a) a view of subject that focused on connections and meaning, (b) a curriculum focused on important mathematical ideas, (c) a balance of coordination and professional discretion for teaching decisions, and (d) clear distinctions between doing math and doing school in the structures of the classroom and the evaluation of students' thinking.

In the remainder of this article, I provide the details of these four common features, including examples from each school. I then discuss the implications of this analysis for departments wishing to undertake detracking reforms.

A Connected and Meaningful View of the Subject

In popular culture, mathematics suffers from a reputation as the most painful and useless of academic subjects. On the comics page, math classrooms provide the standard context for showcasing school-aged characters' humorous angst and boredom. In the classroom, many teachers strain to answer students' perennial question, "When are we ever going to use this?" Despite this tarnished reputation, the teachers at Phoenix Park and East High managed to present a version of the subject that students found meaningful and engaging. At Phoenix Park, 75% of students who were interviewed reported using school mathematics in their daily lives, compared to none of the students taught in the traditional classrooms. Likewise, East High students frequently referred to mathematics as a kind of language, as stated by this senior:

> Math seems like a second language or another language that we're learning—because it is something that you can use to communicate to others through math. (Boaler, 2004)

This student's view of the utility of mathematics was common among students at both schools.

How are the Phoenix Park and East teachers imparting a perspective of mathematics to their students that is so divergent from popular conceptions? In part, it stems from their own views of the subject, which differ from what we typically find in our schools. Many math teachers in the United States and England have what is referred to as a *sequential view* of the subject (McLaughlin & Talbert, 2001; Ruthven, 1987; Stodolsky & Grossman, 2000). That is, they regard mathematics as a well-defined body of knowledge that is somewhat static and beholden to a particular order of topics. This perspective has logical consequences for instruction and student learning. First, in light of this view, the main goal of teaching is to cover the curriculum in sequence to achieve content goals. Second, students must master prior topics in the sequence to move forward in the curriculum successfully.

This view of mathematics makes detracking appear less feasible. Gaps in students' prior learning are seen as obstacles to their present learning, making divisions between low-achieving and high-achieving students a necessity. Teachers at Phoenix Park and East High School had a more connected and multifaceted view of the subject (Hiebert et al., 1996; Resnick, 1988). They manifested this view through the use of open-ended problems as the basis for their curriculum.

Making sense of mathematics at Phoenix Park. At Phoenix Park, the teachers directed students' mathematical investigations in a deliberate way. As Boaler (2002a) reported, they:

> did not subscribe to the common belief that lower attaining students needed more structure. They merely asked different questions of the students to help them make the connections they needed to make. (p. 168)

In this description, the teachers' conception of mathematics appears different than the image of hierarchically organized topics. Instead, mathematics is a network of interrelated ideas with connections that can be understood by students with different levels of attainment, given appropriate and differentiated scaffolding. The teachers used problems that required students to make meaning of the mathematics they were using, as they had to clarify assumptions and explore and defend their choices in problem posing and problem solving. Boaler (2002a) found that Phoenix Park students performed more sensibly and creatively on an open-ended design task (designing an apartment that fits certain mathematical criteria) than students who had received traditional instruction. For the Phoenix Park students, mathematics was a tool they brought to bear on problems in the world, not just a set of procedures whose meaning was bound up in school.

Valuing careful thinking over speed at East High School. At East High School the teachers shared a similar conception of mathematics. In the following excerpt from a department meeting, East math department cochair Guillermo Reyes advised a new teacher who was struggling with a perceived gap between the fast and slow students in her classroom:

> The [students] that are moving through things really quickly, often they're not stopping to think about what they're doing, what there is to learn from this activity. A kid knowing, "Okay, I can get through this quickly but I'm working on X—being a better group member because it's going to help me in my future classes. Showing off math tools because I know how to do it with a t-table[4] but I don't know how that relates to a graph yet."
>
> But like think of the ones that you think of as fast learners and figure out what they're slow at.

> (East High School Algebra Meeting, September 21, 1999)

Although mathematics was not discussed at length, a distinctly nonsequential view of mathematics undergirded Guillermo's statements. In Guillermo's talk, mathematics was a subject with connections: He imagined a student needing to connect t-tables and graphs. More subtly, Guillermo's reworking of the novice teacher's categories of fast and slow students ties in with notions of mathematical competence. Because students, in his terms, are not simply fast or slow learners of mathematics, the subject itself takes on more texture. Mathematics competence is not simply the mastery of procedures—something with which students are more or less facile. Instead, because mathematics is viewed as a connected web of ideas, knowing mathematics requires careful consideration of the various facets of any particular concept and the identification of the relationships among them. Guillermo revealed this last view of mathematical competence when he expressed concern about "the ones who move things really quickly … not stopping to think about … what there is to learn from this activity." To learn mathematics, in other words, students must make sense of mathematics, not simply complete their work to get it done.

The need for sensemaking. The complex and connected view of mathematics shared by both groups of teachers was fundamental to their practice. It implicated the kind of professional knowledge they sought to develop, creating a need for deeper, instead of simply more, content. Additionally, it shaped their attitude toward their students' learning and, as discussed in the next section, their implementation of curricula that would support student sensemaking.

A Curriculum Focused on Important Mathematical Ideas

Phoenix Park and East High math teachers designed their lessons to focus on important mathematical ideas. This approach stands in stark contrast to typical American math lessons, which have been found to be remarkably uniform in structure, often taking the form of "learning terms and practicing procedures" (Stigler & Hiebert, 1999, p. 41). The U.S. lesson structure, common in Britain as well, reflects the underlying sequential view of the subject. If success in mathematics requires mastery of prior topics, then the curriculum needs to be carefully sequenced by teachers and thoroughly rehearsed by students so that they may master the material.

In line with their nonhierarchical view of mathematics, the curriculum at Phoenix Park and East High School countered the typical U.S. and British lesson structure. Instead of learning terms and practicing procedures, both schools' math lessons were organized around big mathematical ideas. This was a deliberate strategy, designed to minimize the deleterious effects of low prior achievement.

Projects and investigations at Phoenix Park. A leaflet put out by the Phoenix Park mathematics department embodied this concept-driven curriculum and its connection to detracking:

> We use a wide variety of activities; practical tasks, problems to solve, investigational work, cross-curricular projects, textbooks, classwork, and groupwork. Every task can be tackled by students with widely different backgrounds of knowledge but the direction and level of learning are decided by the student and the teacher. (Boaler, 2002a, pp. 58–59)

At Phoenix Park, the yearly curriculum consisted of four to five topic areas, each of which were explored through various projects or investigations. A topic area might have a title like *Connections and Change* or *Squares and Cubes*. Boaler (2002a) provided a detailed description of one teacher's introduction to a fairly representative Phoenix Park math project called *36 Pieces of Fencing* (pp. 51–54). In the task, students are asked to find all the shapes they can make with 36 pieces of fencing and to then find their area. This single open-ended problem took up approximately 3 weeks of class time. At Phoenix Park, the teachers used mathematically rich and open-ended curriculum to differentiate their instruction. Although the teachers strongly believed that all students should have access to challenging mathematics, their activities provided different entry points for different students (Boaler, 2002a, p. 57). Problems like *36 Pieces of Fencing* supported a range of mathematical activity. Students could investigate the areas of different shapes, collect data on and construct graphs of the relations between shape and area, explore combinatorial geometry, or use trigonometry. If students finished their work or became bored, the teachers would extend the problems to support their continued engagement.

Group-worthy problems at East High School. Similarly, East's math teachers organized their detracked curriculum around what they called group-worthy problems. In their meetings, the teachers consistently invoked group-worthiness as the gold standard by which classroom activities were evaluated. In one conversation, they collectively defined group-worthy problems as having four distinctive properties. Specifically, these problems: (a) illustrate important mathematical concepts, (b) include multiple tasks that draw effectively on the collective resources of a student group, (c) allow for multiple representations, and (d) have several possible solution paths.

East High math teachers also organized their curriculum into large topical units. For example, one unit called $y = mx + b$ focused on the connections between the various representations (tables, graphs, rules, patterns) of linear functions, connections that are essential to the development of conceptual understanding (Ma, 1999; Schoenfeld, Smith, & Arcavi, 1993). Their units were subdivided into a collection of related activities, all linked back to an overarching theme.

A typical activity in an East High algebra class was *The Vending Machine*. In this problem, students were told about the daily consumption patterns of soda in a factory's vending machine, including when breaks were, when the machine got refilled, and the work hours in the factory. Students were then asked to make a graph that represented the number of sodas in the vending machine as a function of the time of day.

The activity focused on one larger problem organized around a set of constraints. Although these constraints limited the possible answers, students had an opportunity to discuss the different choices that would satisfy the constraints and look for common features of plausible solutions as a way of generalizing the mathematical ideas. Embedded in the activity were important mathematical ideas (graphing change, slope, rate) that were linked to a real-world context.

Interpreting the world through mathematics. The two curricula had in common an approach to teaching mathematics through activities that required students to use mathematics to model and interpret situations in the world. These curricular approaches were aligned with the view of mathematics as a tool for sensemaking: Students need opportunities to understand mathematics through activities that allow them to make sense of things in the world. Although there were differences in the execution—there was more latitude for curriculum differentiation in the Phoenix Park curriculum and more structured group work at East High—the conception of mathematics that they shared allowed the participation of students of varied prior preparation.

A Balance of Professional Discretion and Coordination for Teaching Decisions

Detracked classrooms may make it harder for teachers to proceed through the curriculum in a lockstep fashion. Detracking increases the urgency for teachers to respond to the particularities of the learners in their classrooms. At the same time, teachers need frameworks for decisions about what is important to teach to articulate to the larger curricular goals. Both groups of teachers organized their work to allow for individual adaptation and, simultaneously, a degree of coordination.

At both schools, the teachers collaborated on the development and implementation of their respective curricula. In addition, it is probably not a coincidence that both groups controlled the hiring of new mathematics teachers in their department—a common practice in England but highly unusual in the United States. As a result, both groups of teachers were working with like-minded colleagues. Their shared values surely facilitated the implementation of common frameworks and practices.

Looping through a common curriculum at Phoenix Park. At Phoenix Park, the teachers balanced professional discretion and coordination by keeping a group of students with the same teacher for several years (a practice known as *looping*) while teaching from a common curriculum that they consulted about in an ongoing fashion. The looping structure changed the time that teachers had to work with their students from 1 to 3 academic years, allowing for more adaptations by individual teachers and a more in-depth knowledge of particular students. Looping also minimized the transitions between teachers that can challenge low-performing students (Horn, 2004b).

At the same time, in their math department meetings, the teachers would discuss the activities they planned to use and any modifications they needed to make. These meetings allowed teachers to vet ideas past colleagues and consult on challenges that arose, instead of requiring them to work in isolation. Although the teachers drew on each other's knowledge and experience with their common curriculum, their classrooms reflected their individual teaching styles and managerial preferences.

Coordinating for student learning at East High School. The East High math teachers' course structure required a greater degree of coordination. Students stayed with the same teacher for one term, with the school year consisting of two terms. This meant that students could encounter anywhere from three to seven math teachers during their 4 years of high school, a structure that increased the demand for coordination. As a result, the East teachers had more explicit structures to support this coordination.

At the start of each new academic term, the teachers gathered for what they called a *roster check*. Each teacher brought class lists to show to all the other teachers. In this way, they could alert each other to vulnerable students and share effective strategies for working with them. Additionally, the teachers met weekly in their subject groups (e.g., algebra, geometry) and discussed curriculum and its effective implementation. They worked collaboratively to develop and refine their curriculum, adapting published materials to make them more group-worthy. The teachers also paid close attention to the ways they presented ideas, the kinds of questions asked, and employed language that might make mathematical ideas most meaningful to students (Horn, 2004a). For instance, East's teachers avoided commonly used terms like *canceling out* to describe the result of adding opposite integers such as –3 + 3. Instead, they preferred the phrase *making zeroes,* as it more accurately described the mathematics underlying the process.

At the same time, individual teachers often took their own paths through the common curriculum based on their own judgments about their particular classes' strengths and needs. They did so in consultation with the colleagues who would be teaching the students in subsequent courses.

Common vision, adaptive implementation. Both groups of teachers had structures that supported the student-centered coordination of their teaching. At Phoenix Park, the common curriculum and the department meetings were the main vehicles for coordination. At East High, where teachers' interdependence was increased by their course schedule, a greater number of structures were required: roster checks, weekly subject-specific meetings, and attention to common language. Although their contexts demanded different means for flexibly coordinating practice, both groups of teachers had one thing in common: They effectively used their colleagues as resources for their own ongoing improvement of practice. They had structures in their workweek that allowed them to consult with each other and learn from their collective experience, breaking through the privacy and isolation that often characterizes teachers' work (Little, 1990). This has generally been found to be true of departments that support students' participation in advanced mathematics courses (Gutiérrez, 1996).

Clear Distinctions Between *Doing Math* and *Doing School* for Students and Teachers

One of the effects of ability grouping is that, despite its name, students are placed according to their prior school achievement, not by their potential to learn. In this way, schooling savvy is conflated with mathematical competence. If students know how to turn in homework and study for tests, they will likely be placed in a higher track than equally capable students who have not mastered these school learning practices.

Within two very distinct school contexts, the Phoenix Park and East High mathematics teachers worked to make practices of schooling transparent to their students. The nature of Phoenix Park and East High School themselves afforded different kinds of teaching and learning, and therefore placed different demands on students' schooling know-how. Phoenix Park School, a comprehensive public school with no entry requirements or special charter, had about 600 students. Many of the departments used project-based curricula. The school's progressive philosophy aimed to develop students' independence. In contrast, East High School was a more traditionally configured comprehensive public school of 1,500 students. The subject departments varied widely in their approaches to curriculum and instruction. Within the school, the math department was seen as a leader for many schoolwide reforms, such as the shift to block scheduling and the creation of a peer-tutoring clinic. The two schools brought different resources and challenges to the math teachers' detracking work.

Focusing on student thinking at Phoenix Park. At Phoenix Park, the classrooms were minimally structured, with students electing to work independently or in groups, often socializing in between their pursuit of solutions to their open-ended projects. This complemented the larger school goals of fostering students' independence. Within this open setting, however, the teachers valued particular learning practices and made these standards clear to their students. For example, their teaching approach relied on students explaining their reasoning, thus teachers would frequently prompt students to do so. They paid particular attention to reluctant students, regarding students' difficulties in communicating their thinking or interpreting their answers not as resistance but instead as a gap in the students' understanding about classroom expectations (Boaler, 2002a). In addition, in their progressive setting, the teachers had the liberty to emphasize learning through formative assessments, commenting on the quality of student work without assigning it particular grades. This allowed teachers and students to focus on individual students' learning over their ranked school performance.

Teaching* all *students how to learn at East High. In the traditional comprehensive high school setting of East, the math teachers conducted their classes in a more structured fashion. Although the curriculum was open-ended, the students were expected to work while in class, usually in small student groups. The teachers had received extensive training in a teaching method called complex instruction (Cohen, 1994) that allowed them to use group work as a vehicle for challenging students' assumptions about who was smart at math. They aimed to broaden students' notions of what it meant to be good at math, thereby generating greater student participation and success in the subject.

In line with their goal of increased participation, the teachers were explicit that learning to be a student was an important part of their curriculum, and they came up with structures to support that learning. At the front of each classroom was a homework chart laid out much like a teacher's roll book, with students' names in a column along the side and the number of each homework assignment across the top. Although actual grades were not posted, completion of homework was represented by a dot. The homework chart reminded students of the primacy of homework in their job as students. The teachers and the students could glance at it and see if the students were doing their job. If students did not complete their homework on a given day, they were assigned an automatic lunch or after-school detention. It was viewed as a major coup when the math teachers got the sports coaches to agree to not allow athletes to come to practice on days when they had missed their math homework.

At the same time that they emphasized traditional student skills such as doing homework, they did not confuse failure in class with students' intelligence or ability. In interviews, the East teachers frequently used the following phrase to qualify a student's poor performance: "He was not ready to be a student yet." They worked to convey this mindset to their students too: All East math teachers had a large sign with the word *YET* placed prominently in their classrooms. In this way, when a student claimed to not know something, the teachers could quickly point to the giant *YET* to emphasize the proper way to complete such a statement.

Focusing on students' potential to learn. By making clear distinctions between doing school and doing mathematics, the teachers at both schools focused themselves—and their students—on the students' *potential* to learn. Many of the preceding examples come out of a shared emphasis on formative assessment, activities undertaken by teachers (and students) to provide information and feedback that modified their teaching and learning activities (Black & Wiliam, 1998).

This distinction also allowed explicit conversations about the schooling practices that would help support students' learning and academic success. Given that students at both schools often came from families whose parents had not succeeded in formal education, the teachers' assumption of this responsibility helped to create more equitable classrooms (Delpit, 1988).

Conclusion

This article summarized common features of Phoenix Park and East High School's efforts to detrack their secondary mathematics curriculum. Both schools succeeded in creating more equitable environments for teaching and learning by adopting a complex and connected view of subject, implementing a curriculum focused on important mathematical ideas, finding a balance between coordination and professional discretion for teaching decisions, and making clear distinctions between doing math and doing school in their teaching and in their evaluation of student learning.

These four features of the teachers' practice focus the students and teachers on mathematical thinking, valuing understanding over coverage. Although this has been a theme of current reform efforts in mathematics (National Council of Teachers of Mathematics, 2000), figuring out how to operationalize slogans like *teaching for understanding* is a challenge when teachers have not had opportunities to develop understanding themselves; are pressed toward the competing goal of curriculum coverage; work in isolation from their colleagues; and work in systems that value summative over formative assessments. The work of the Phoenix Park and East High teachers demonstrates how two groups of teachers managed these tensions in their own school and classroom settings.

It is important to emphasize that although this analysis provides a list of common characteristics of the two departments, it does not do justice to the complex ecologies of these places and the synergistic way that these features interacted. For example, an open-ended curriculum on its own cannot induce the equitable outcomes that resulted at Phoenix Park and East High School (Boaler, 2002b). If teachers' view of the subject is dominated by the importance of mastering topical sequences and overlooks the deep connections across topics, they may hesitate to wade into the murky territory of open-ended problems. Or, if teachers' efforts to coordinate the implementation of open-ended problems lead them to thwart professional judgments about their own strengths and needs or those of particular students, they may, for instance, follow another teacher's pace in a manner that is inappropriate to their own classrooms. Likewise, if teachers do not make schooling practices explicit to students who are attempting open-ended work, they will encounter resistance—or, as the Phoenix Park teachers would view it, an unproductive gap in students' understanding of what it takes to be successful.

As educational research continues to document further cases of successful detracked mathematics departments, this list can be refined and expanded. In the meantime, it can serve as a provisional guide for schools undertaking the challenge of detracking their mathematics departments.

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Notes

1. All proper names of schools and teachers are pseudonyms.
2. In studies emerging from Boaler's 4-year longitudinal study of mathematical teaching and learning, she refers to East High School as "Railside High School." My postdoctoral work, for which Boaler served as a mentor, linked my studies of the math department's teacher community with her study of mathematical teaching and learning. We have received participants' permission to link these data sets.
3. At Phoenix Park, students significantly outperformed students taught in traditional classrooms on the national exam and other mathematics assessments administered for the study. When Boaler looked at students' performance on procedural and conceptual questions, both groups of students performed equally well on procedural questions but the Phoenix Park students performed better on the conceptual questions, demonstrating a deeper understanding of the mathematics that was taught. At East High School, students entered at significantly lower mathematics achievement levels than the students in suburban schools, but within 2 years they were outperforming the other students, scoring at significantly higher levels on mathematics tests that were tied to the curriculum. In addition, on questionnaires and in interviews, students from both schools reported enjoying mathematics more and finding it more applicable to their daily lives.
4. The East teachers refer to in–out tables used for functions as t-tables.

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